Experimental Vibrational Mechanics – Basis

K-S Chen

Dec., 2021

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Agenda

- A general introduction on vibration testing
- Vibration testing instruments
- Diagnostics
- Vibration control

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Good Textbooks

- K. G. McConnell, *Vibration Testing, Theory and Practice*, Wiley 1995.
- Beckwith, Marangoni, and Linhard, *Mechanical Measurements*, 5th Ed. Addison Wesley, 1993.

Part I: A general introduction on vibration testing

Purpose of Vibration Testing

- Machine diagnostics
  - Failure analysis
  - Performance evaluation
- System dynamics extraction
  - Obtaining dynamic parameters
  - Construct mathematical expression for systems

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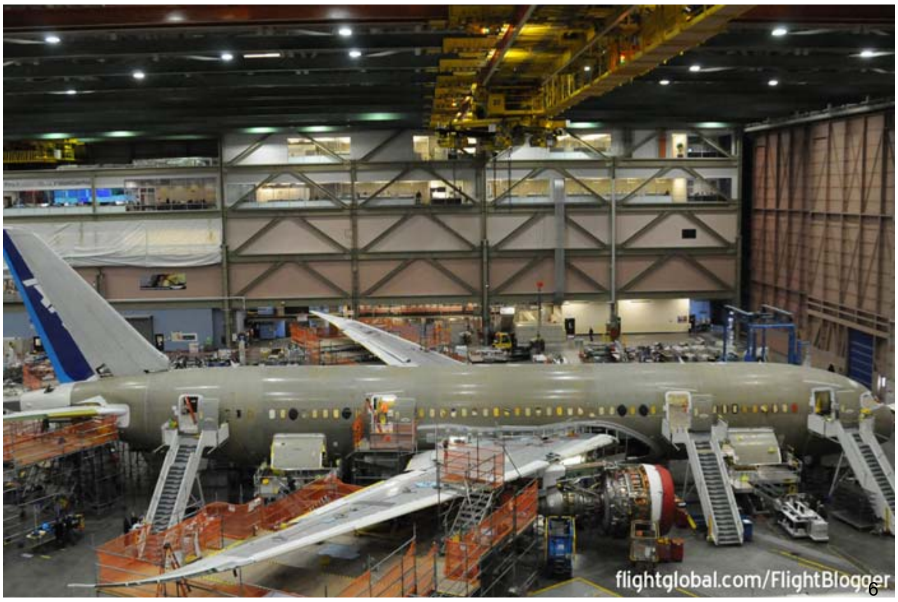
Example: Airplane Testing



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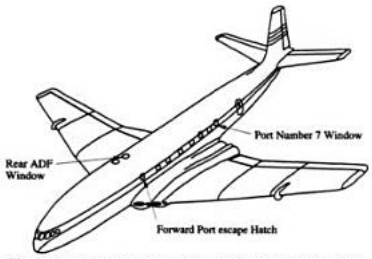

Fig. 3. The main failure sites on Comets G-ALYP and G-ALYU.

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Example: Electronics Reliability Testing

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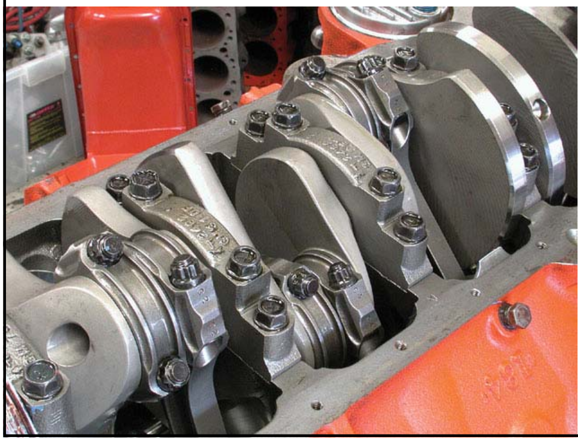
Example: Balancing of rotors and Shafts



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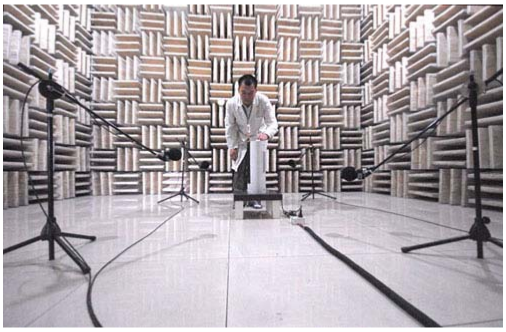
Example: Noise Measurement



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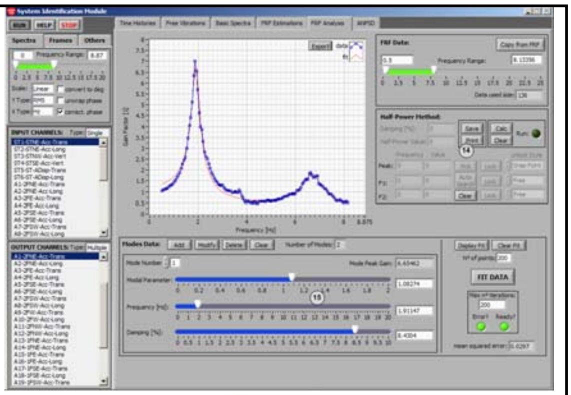
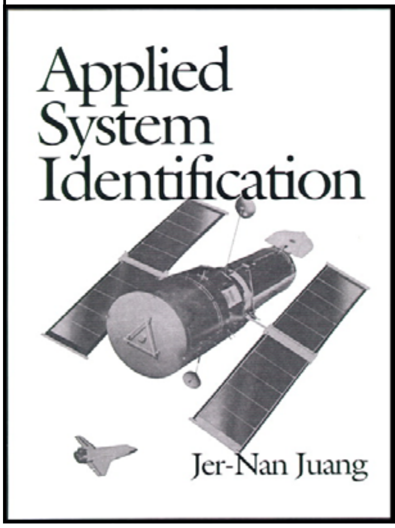
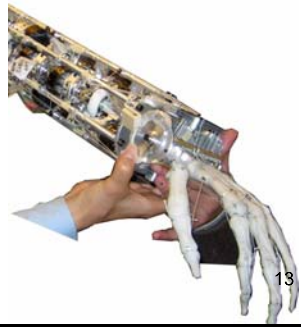
Example: System Identification for Control







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Example: Vibration Isolation

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Testing procedures

- Introduction
- Basic block diagrams and FRF
- Swept sine procedures
- Random signal input

Modal Analysis

- 以實驗方式決定結構體之自然頻率與振動模態
  (Determining natural frequencies & modes via experiment)

Procedure:

1.連續(continuous)結構

⇩

2.離散(discrete) 結構

⇩

3.激發與量測 (excitation & measurement) :
excitation-衝擊鎚(hammer)、振動器(shaker). measurement- 加速規 (accelerometer), 力規 (load cell), 雷射速度儀 (Laser velocity meter)

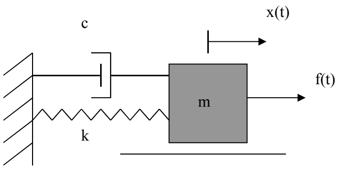
⇩

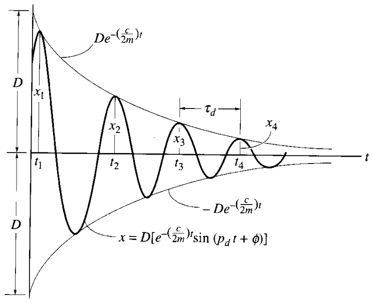
4.DAQ & Processing: spectrum analyzer(e.g.,HP3566A、HP VXI system, etc)

⇩

5.Modal analysis: ME’Scope , STAR

Basic Vibration Model





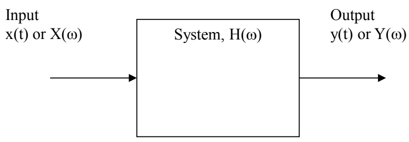
$$m\ddot{x} + c\dot{x} + kx = f(t)$$

$$\omega_n = \sqrt{\frac{k}{m}}$$

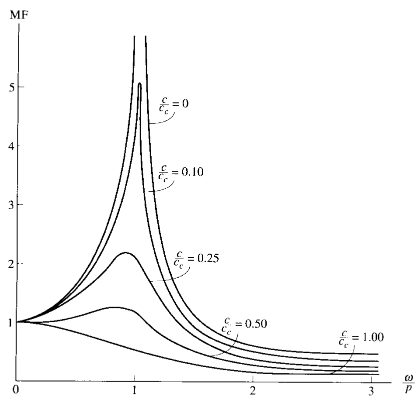
It is possible to find ω_d, and c from time responses

Frequency Response

- Similar to Bode Plot
  - Solved by Laplace transform
- Frequency response function (FRF)



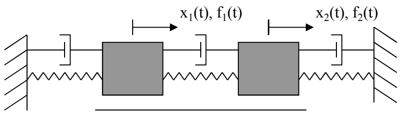
$$H(\omega) = \frac{Y(\omega)}{X(\omega)}$$



Natural frequency, damping ratio can be obtained from frequency response curves

Multi-Degree of Freedom System

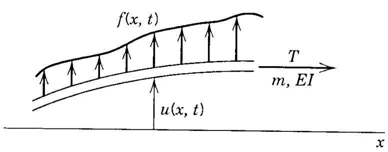
- A series of differential equations
  - Can be modeled by FBD or energy methods
- Mass matrix, stiffness matrix
- Eigenvalue
  - Natural frequencies
- Eigenvector
  - Mode shapes



$$\begin{Bmatrix} x_1(t) \\ x_2(t) \end{Bmatrix} = \begin{Bmatrix} X_1 \\ X_2 \end{Bmatrix} e^{i\omega t}$$

Continuous Systems

- Governing by partial differential equations
  - E.g., plate, bar, string, and general structures
  - E.g. Simple Euler Bernoulli Beam
  - Equation can be obtain
    - FBD
    - Variational formulation



$$\underbrace{m\frac{\partial^2 u}{\partial t^2}}_{\text{inertia}} + \underbrace{C\frac{\partial u}{\partial t}}_{\text{damping}} \underbrace{\overbrace{- \frac{\partial}{\partial x}\left[T\frac{\partial u}{\partial x}\right]}^{\text{tension}} + \overbrace{\frac{\partial^2}{\partial x^2}\left[EI\frac{\partial^2 u}{\partial x^2}\right]}^{\text{bending}}}_{\text{stiffness}} = \underbrace{f(x, t)}_{\text{excitation}} \qquad (3.8.1)$$

Example Free-Free Beam

$$\ddot{\eta}+(\lambda^4\frac{EI}{m})\eta=\ddot{\eta}+\omega^2\eta=0$$

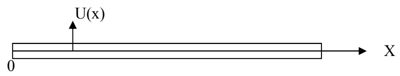
$$\frac{d^4U}{dx^4}-\lambda^4U=0$$

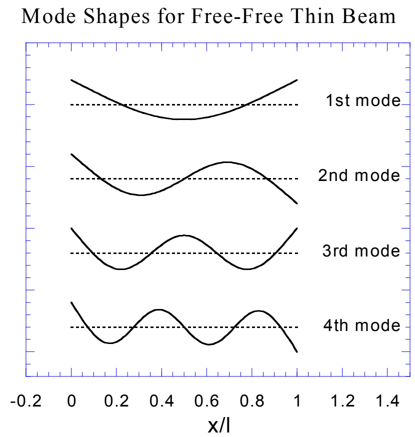
$$f_1=\frac{\omega_1}{2\pi}=\frac{1}{2\pi}\sqrt{\left(\frac{4.730}{l}\right)^4\frac{EI}{m}}$$

$$f_2=\frac{\omega_2}{2\pi}=\frac{1}{2\pi}\sqrt{\left(\frac{7.853}{l}\right)^4\frac{EI}{m}}$$

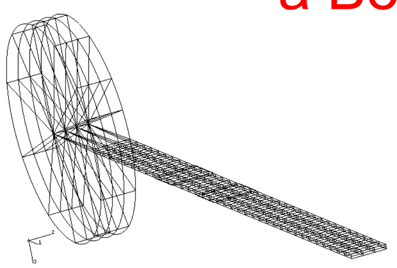
$$f_3=\frac{\omega_3}{2\pi}=\frac{1}{2\pi}\sqrt{\left(\frac{10.996}{l}\right)^4\frac{EI}{m}}$$

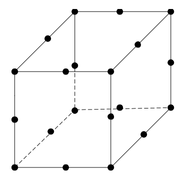
$$f_4=\frac{\omega_4}{2\pi}=\frac{1}{2\pi}\sqrt{\left(\frac{14.137}{l}\right)^4\frac{EI}{m}}$$

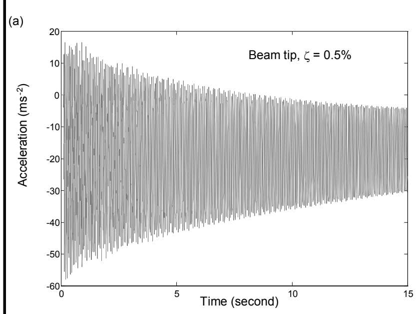


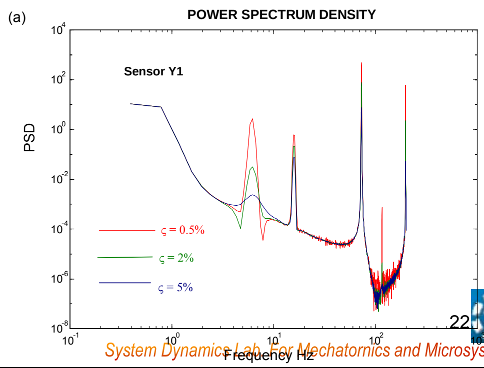


Example: FEM Analysis for a Beam-Hub Structure

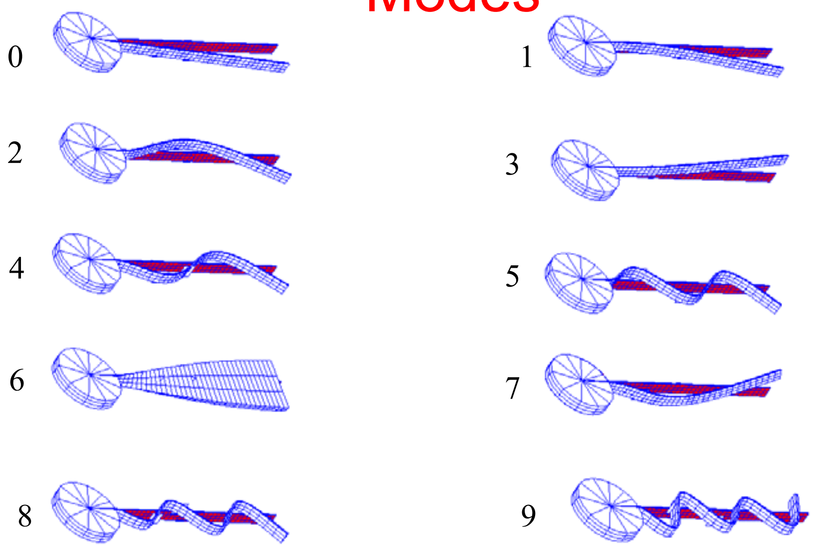




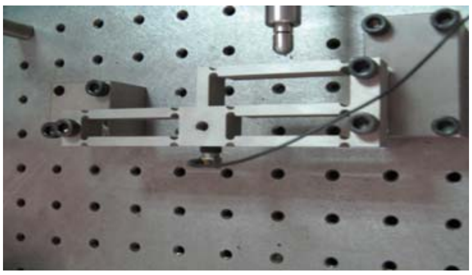


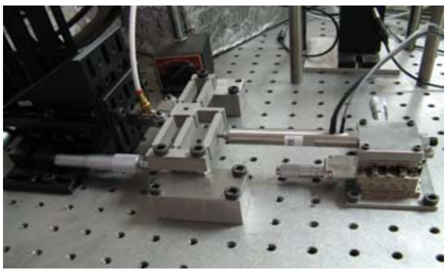


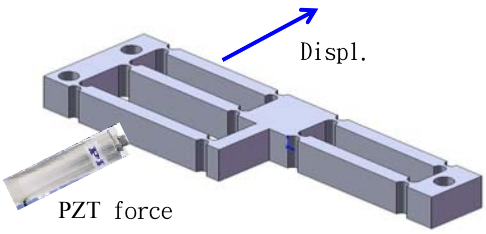
Rigid Body & the first 9 Structural Modes



Example: Compliant Stage Design

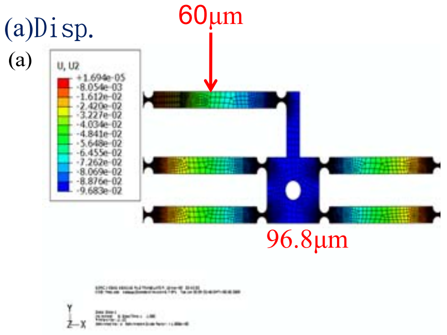


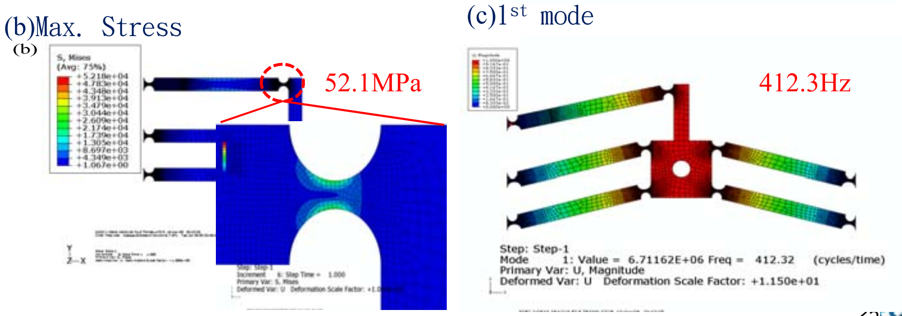




Finite Element Analysis

E=68.9GPa
ρ=2700kg/m³
poison ratio=0.33





Rotor Dynamics (I)

- Natural frequencies varies with rotating speed
- Forward whirling
- Backward whirling
- Important for vibration reduction for high speed rotating objects
  - Shaft
  - Disk,…

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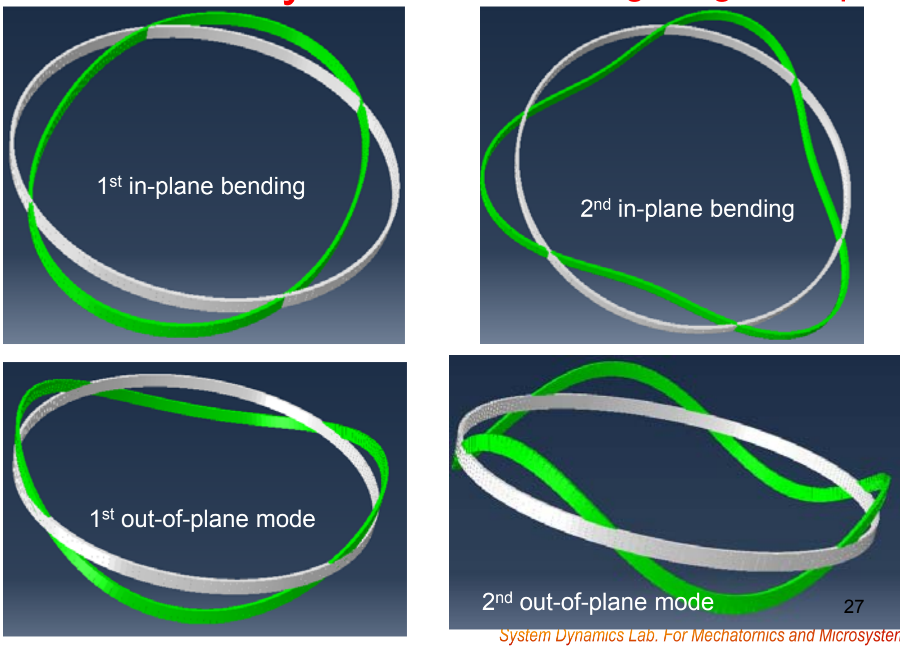
Rotor Dynamics: Rotating Ring Example



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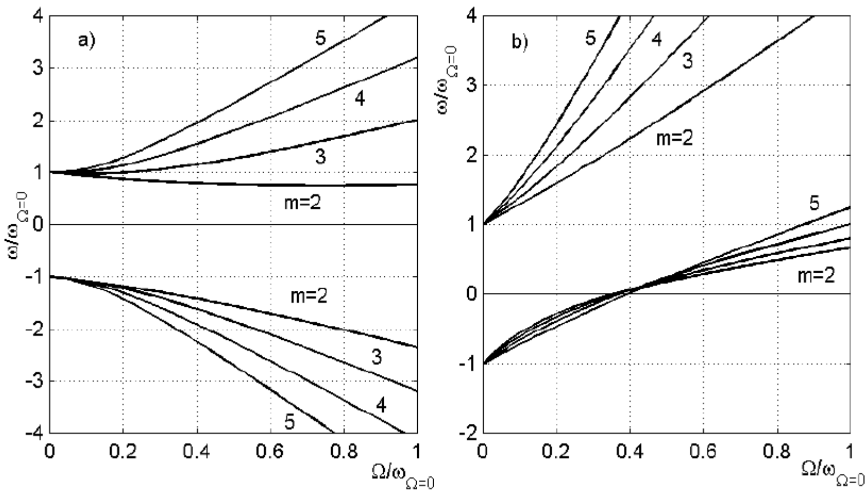
Rotor Dynamics: Campbell Diagrams



Rotating Frame

Fixed Frame

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Swept Sine Procedure

- Use single frequency input signal to excite system (swept from low to high frequency)
- Waiting for the transient die out
- Recording the output magnitude and phase
- Construct FRF or bode plot
- Use system identification to find the transfer function or system dynamics
- Usually slow but accurate for low frequency response

Random / Impulse Procedure

- Based on "impulse" response
- Provide a "pulse" excitation to generate the output spectra
- Due to presence of noise, multiple experiments and averaging must be performed
- Usually fast but inferior for low frequency response

Part II: Vibration testing instruments

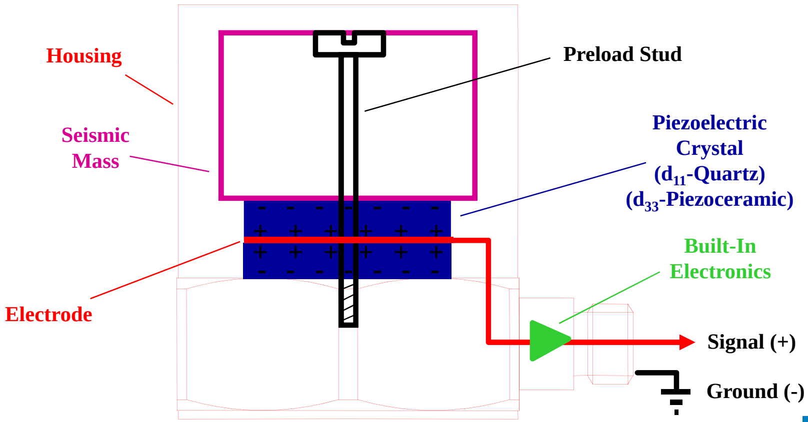
Vibration Test Equipnment

accelerometer

Hammer

Piezoelectric Sensor Structures

- Compression Mode Accelerometer



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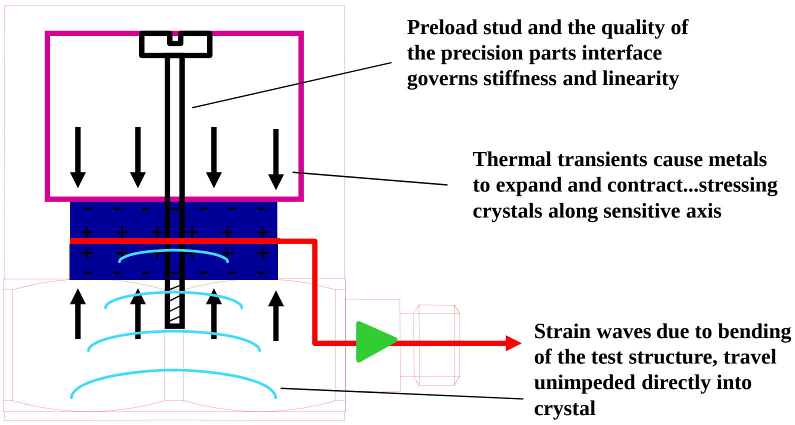
Piezoelectric Sensor Structures

- Compression Mode Accelerometer



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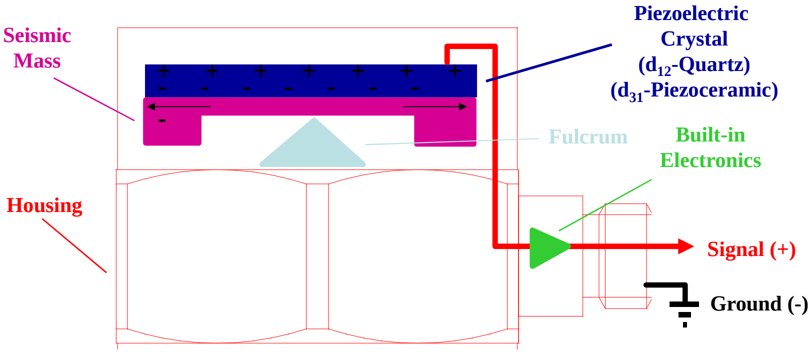
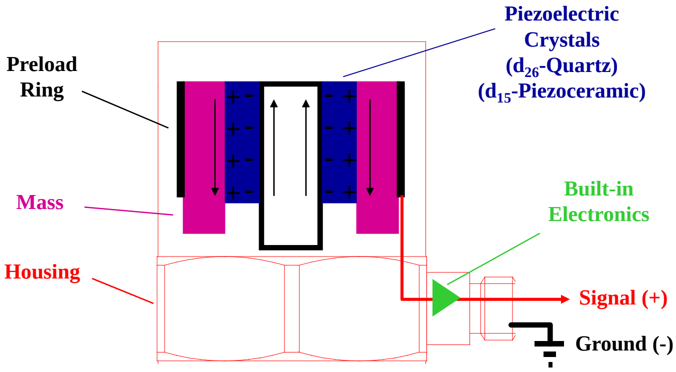
Piezoelectric Sensor Structures

- Beam or Flexural Mode Accelerometer

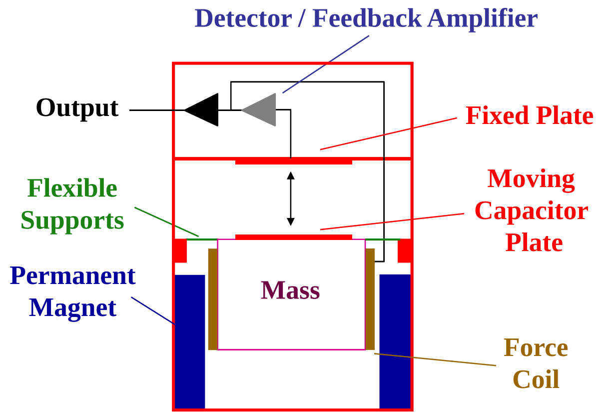


Piezoelectric Sensor Structures

- Shear Mode Accelerometer



Servo / Force Feedback



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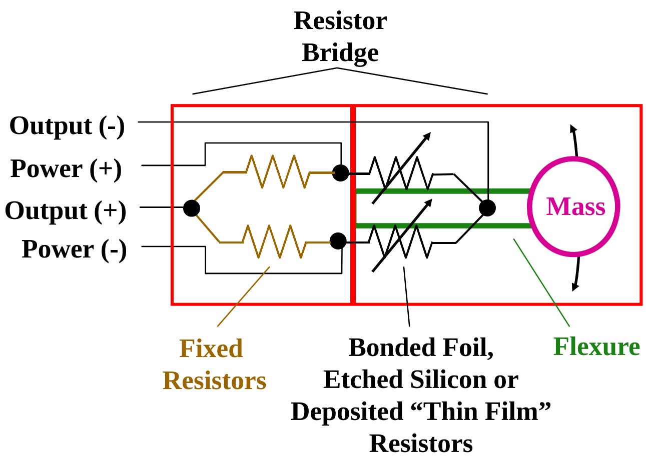
Resistive Type Accelerometer



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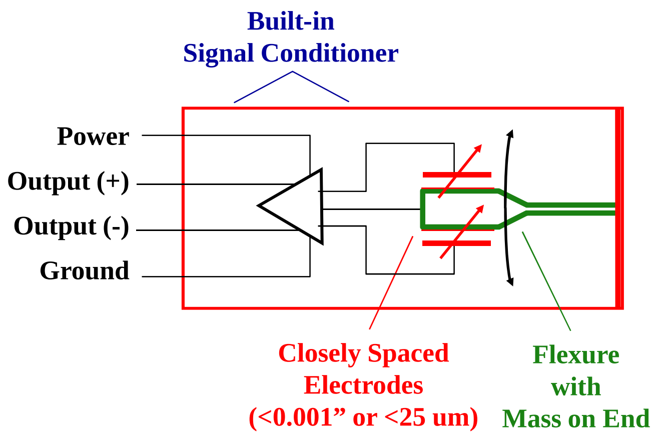
Capacitive Type Accelerometer



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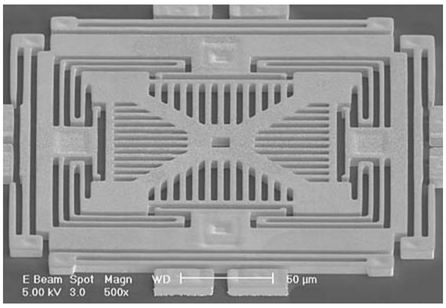
MEMS Accelerometers



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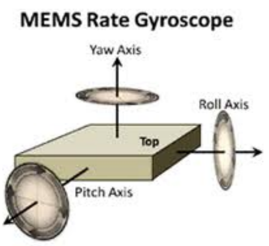
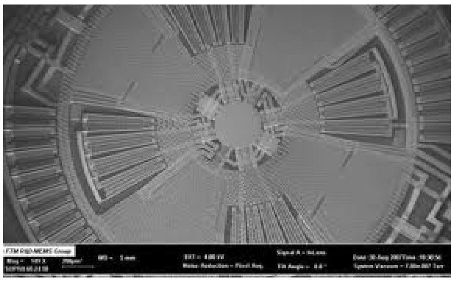
Gyroscope (Angular Rate Sensor)





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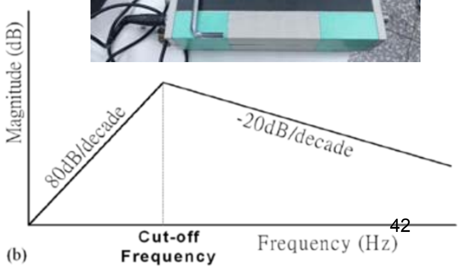
積分器 (Integrator) I

- Work with inertia sensors
  - Acceleration → velocity → displacement
  - Angular velocity → angle
- Not as trivial as you think
  - Need to avoid signal drifting due to near DC component



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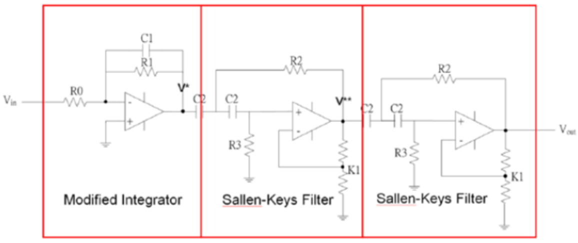
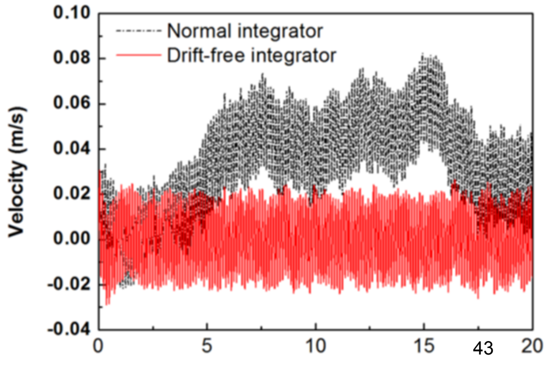
積分器 (Integrator) II





(a)

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Vibration Test Equipnment

- Shaker

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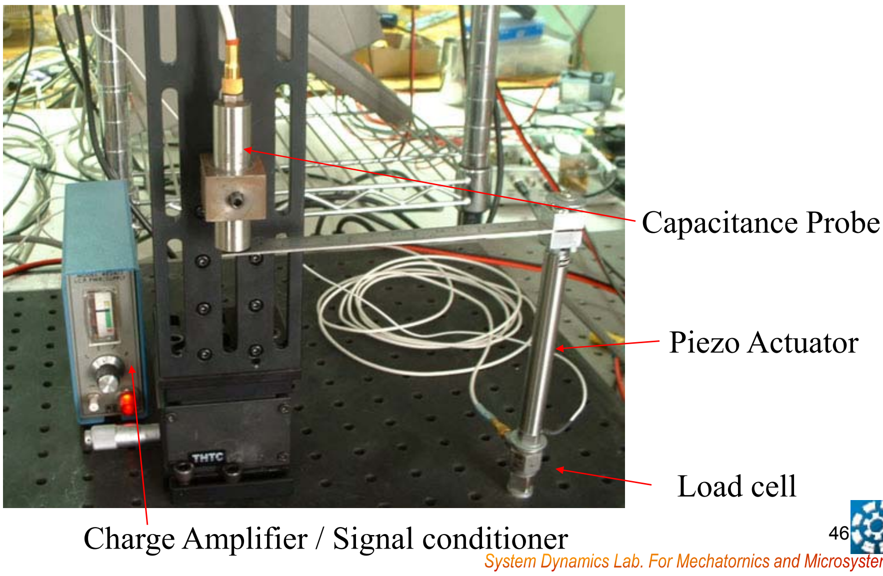
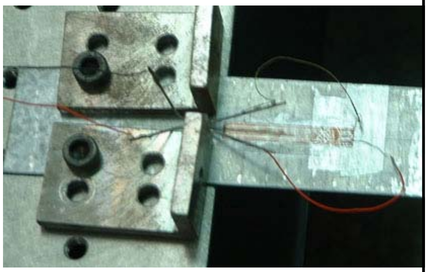
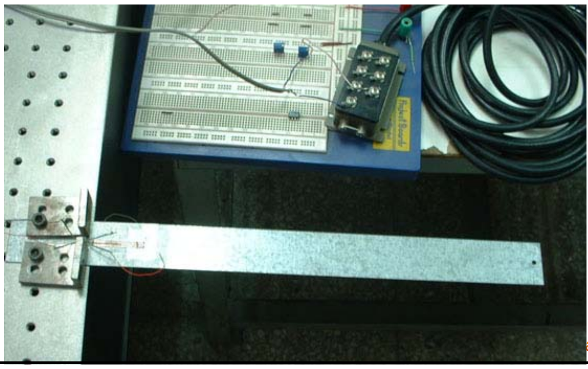
Vibration Test Equipment



Vibration Test Equipment

- Strain gauge





Vibration Test Equipment

Microphone

Accelerometer calibration fixture

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Vibration Test Equipment

- Laser Displacement / Velocity sensor

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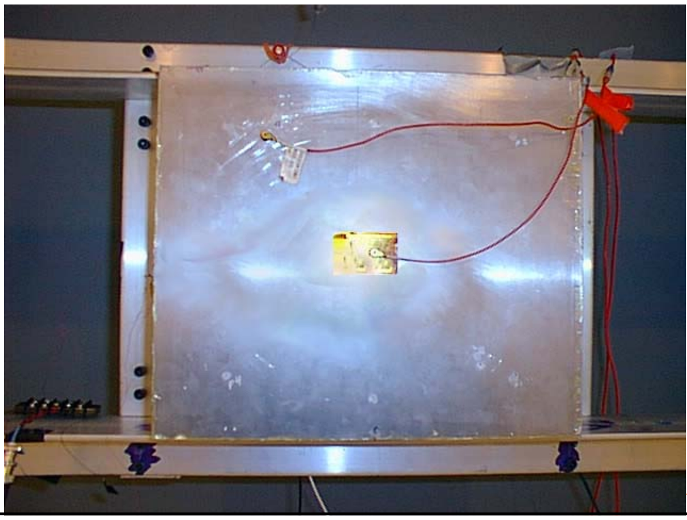
Piezoelectric Actuator and Accelerometer on a Rectangular Plate



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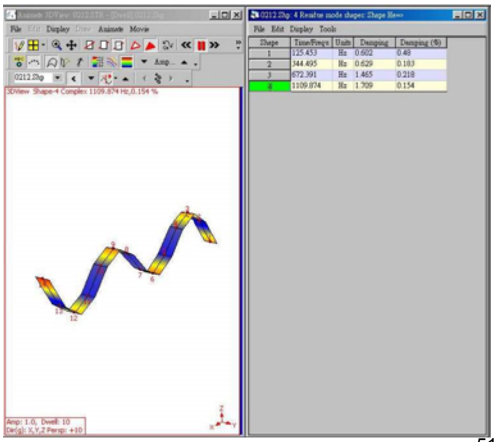
and Microsystems

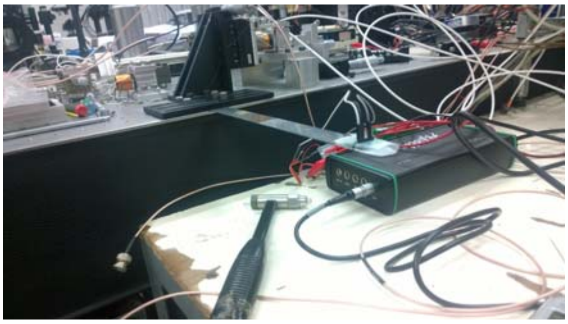
Vibration Analysis Tool

MESCOPE
calculate natural frequency and modes from experimental data

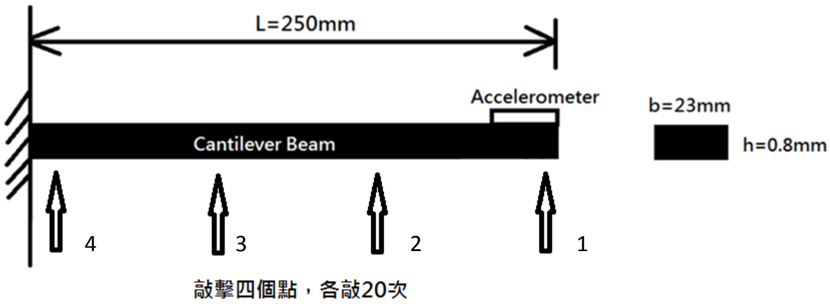


outline

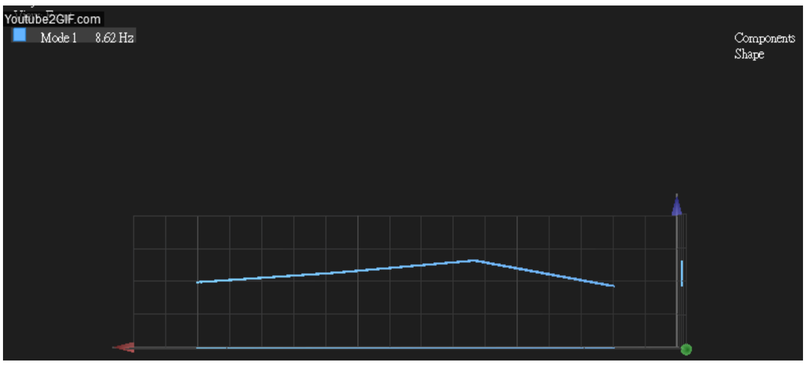
- 使用衝擊錘敲擊鐵尺，使衝擊錘之施力為input，鐵尺末端加速度為output，並使用Apollo和STAR找出transform function和模態。



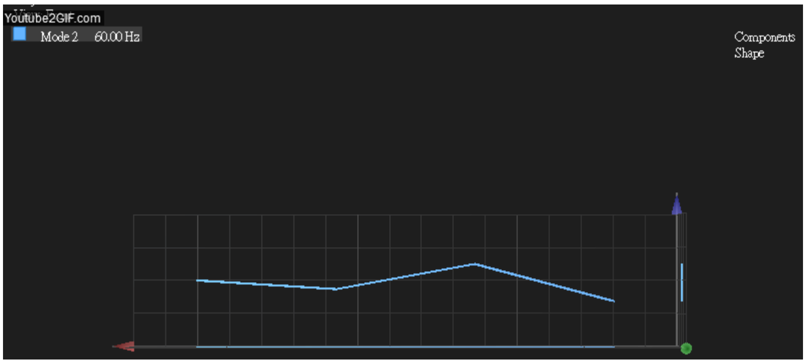
示意圖


L=250mm
Accelerometer
b=23mm
Cantilever Beam
h=0.8mm
4
3
2
1
敲擊四個點，各敲20次

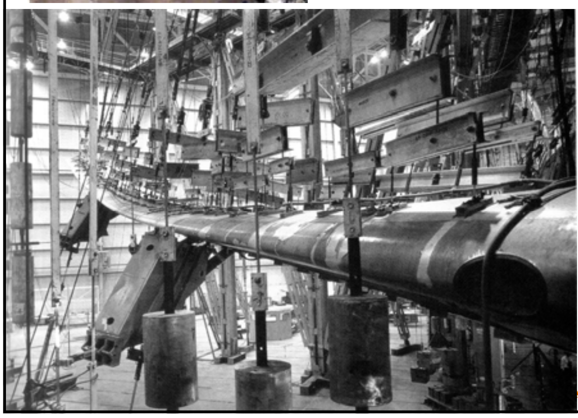
第一模態


Youtube2GIF.com
Mode 1 8.62 Hz
Components
Shape

第二模態



Very Large Scale Application -Boeing 747-



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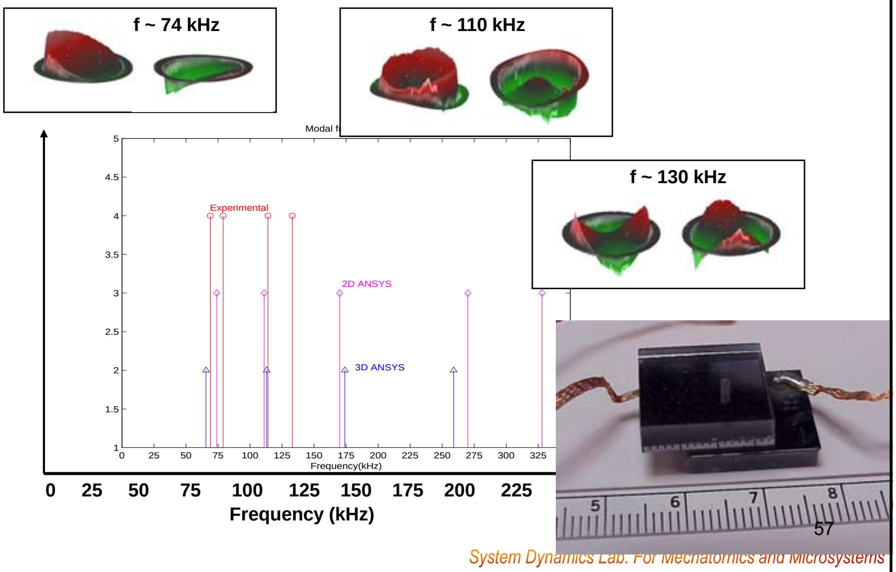
Very Small Scale Application

-Micro Pump-



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Part III:Diagnostics

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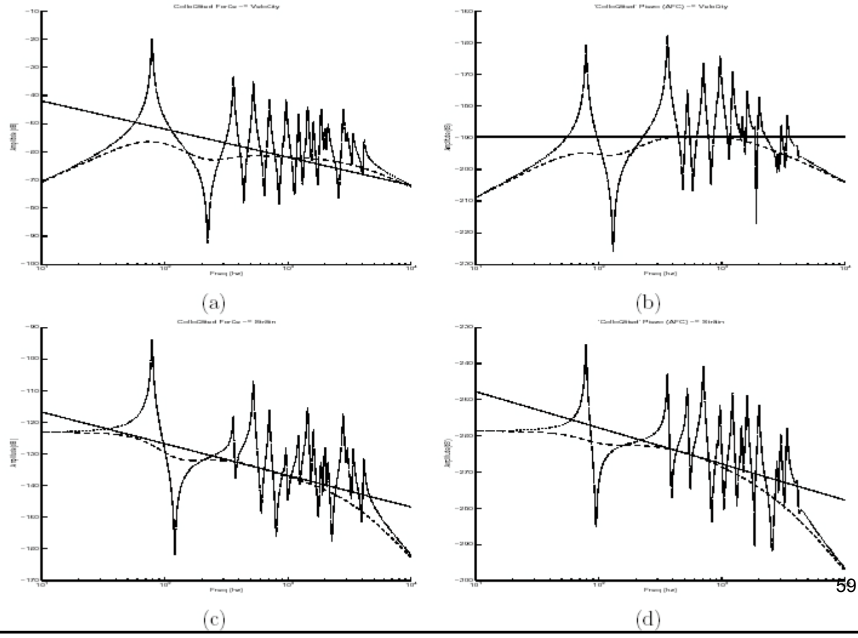
Typical FRF



Data analysis

- Noise
- Windowing
- Averaging
- FRF and system identification

Mass Loading Effect During Measurement

- Mass of accelerometers will change the system dynamics
- May need to use Dunkerley's equation to exclue the effect of mass loading

Noise

- Johnson noise
- Quantization noise
- Parasitic fields
- EM radiation
- Environment vibration

Windowing

- Why windowing?
  - Due to leakage during FFT analysis
- Type of windows
  - Rectangular
  - Hanning
  - Kaiser-Bessel
  - Flat top

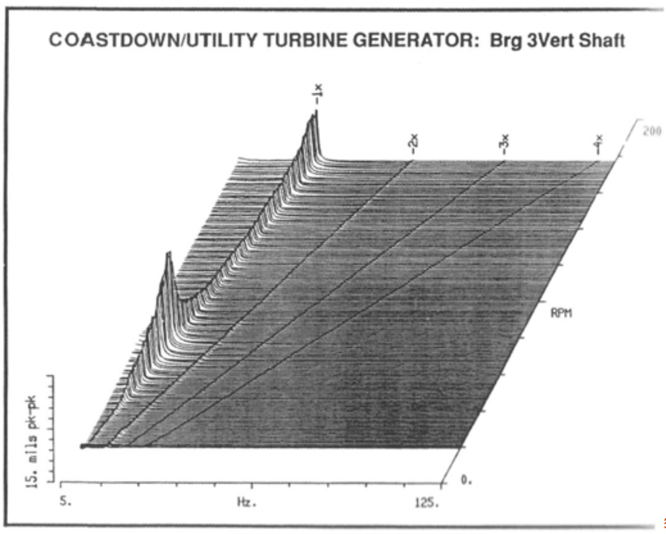
Averaging

- Why averaging?
  - To remove measured random noise
  - It is assumed noise in each event is uncorrelated and the average will be zero if the sample number is large enough
- Averaging algorithms

System Identification

- A process to systematically extract parameters of the transfer function
- May need to guess a form of polynomials for both denominator and numerator
- Use least square, maximum like hood, or Markov chain methods to find the required coefficients
- System model reduction

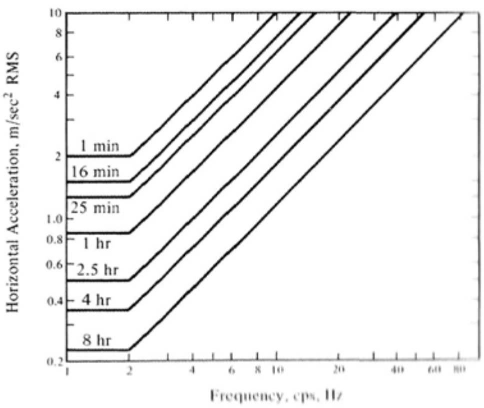
Waterfall or Campbell Diagram



Vibration Diagnostics

- Machine design vs. acceptable vibration level
- Vibration system synthesis
- Imbalance and Rotor dynamics
- Case Study (Gear)

Acceptable Vibration for Human



Horizontal

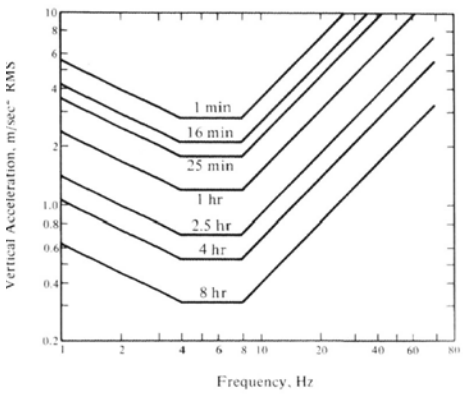


Figure 13.1 ISO—suggested acceptable vertical vibration levels.

Vertical

Acceptable Machine Vibration

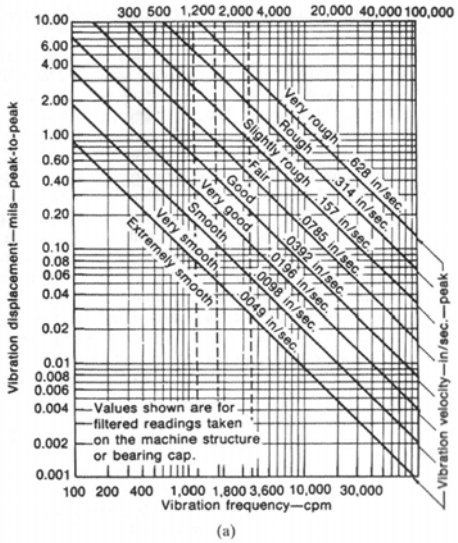


Figure 13.3 Acceptable machinery vibration levels. (Adopted a) from product documentation of IRD Mechanalysis, Inc., 6150 Huntley Road, Columbus, Ohio, b) from ANSI Standard 19–1975.)

Gear Train Spectrum

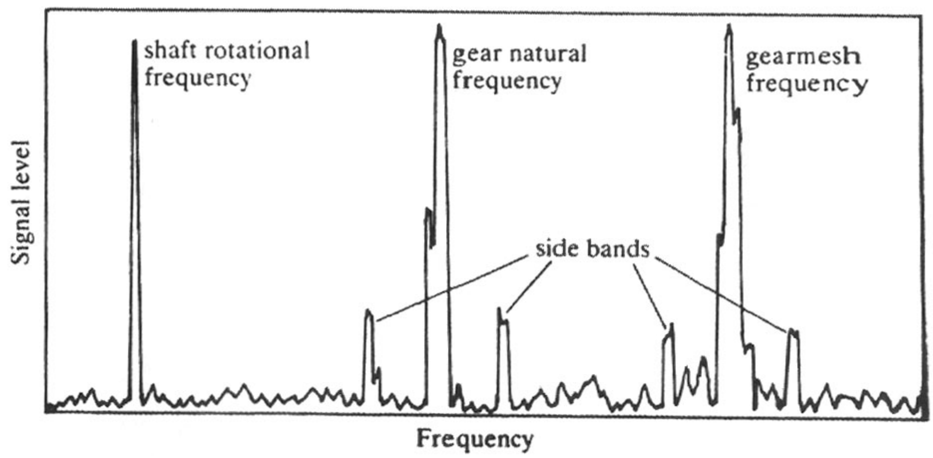


Figure 14.24 Typical gearbox vibration spectrum.

Gear Analysis (Time Domain)

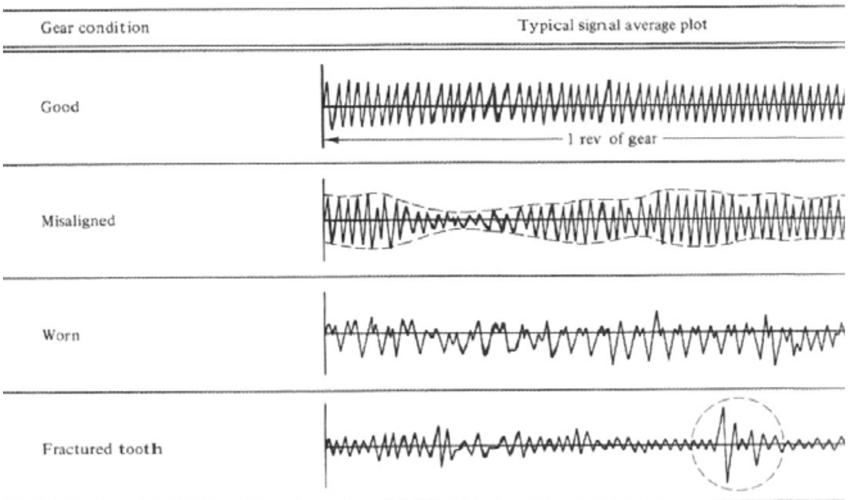


Figure 14.2 Time-series averaging of time-domain gearbox signals.

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Cracked Shaft Spectrum

Edge cracks results in dissimilar stiffness along and perpendicular to the crack tip

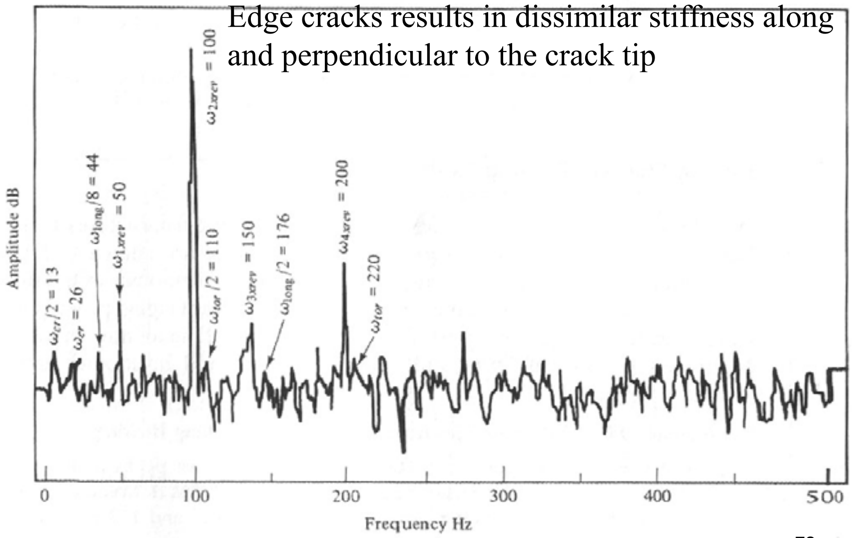

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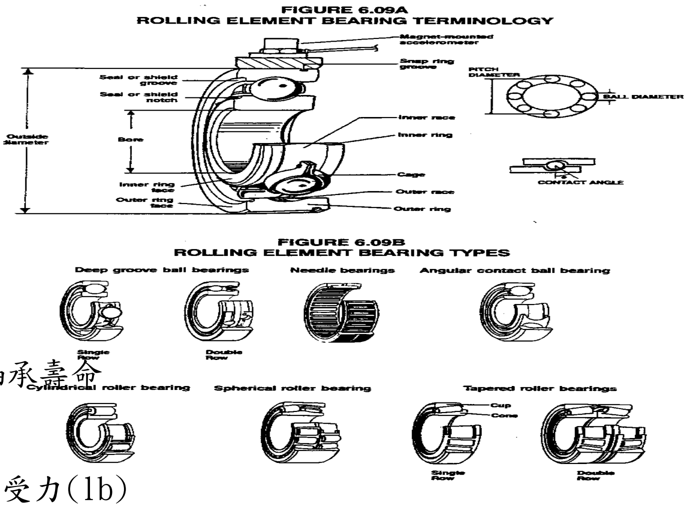
Figure 14.21 Vibration spectrum of a cracked shaft.

Bearing Damage Analysis



L_{10} Life ： 軸承壽命 (hr.)
RPM：轉速
Rating：靜態受力(lb)
LOAD：外力負載(lb)

$$L_{10}\ \text{life} = (16.666/\text{RPM}) * (\text{Rating}/\text{LOAD})^3$$

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Rolling Bearing Damage Analysis

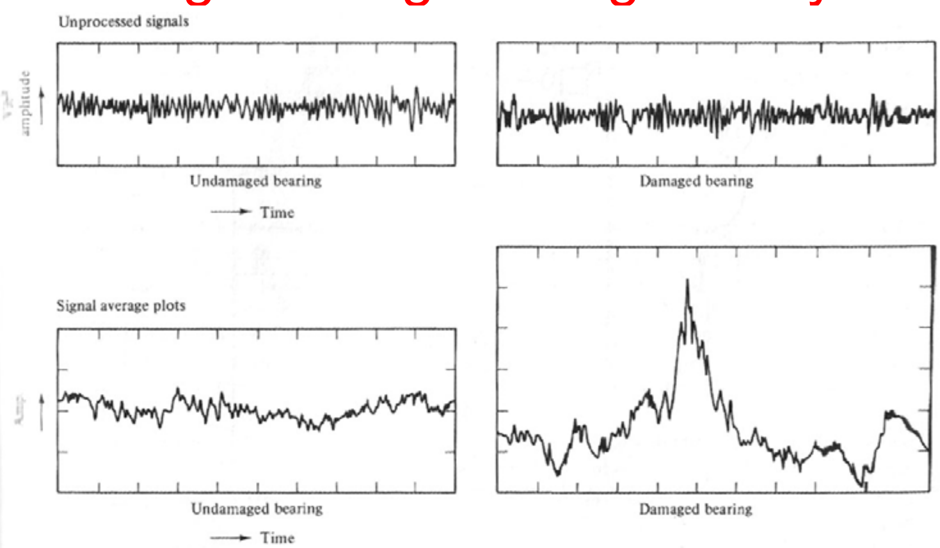

Figure 14.3 Typical signal-average plots for rolling element vibration.

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Spike Energy

- 於一台機台上，轉動的轉子會產生一連續來回的振動能量，此振動能量傳遞到表面上而形成一微弱的振動訊號，通常的轉動零件指軸承或齒輪可藉由特殊的感應器去量測這微小的振動量進而判斷軸承或齒輪的運轉狀況，在表面上所量測到的微小振動量則稱為SPIKE ENERGY以GSE.　gSE為振動單位,可作為早期發現軸承或齒輪損壞的判斷依據。

Possible Source for Causing Spiking

- 軸承磨損(Wear)
- 軸承潤滑不良(Lubrication)
- 轉子或軸封摩擦(friction)
- 齒輪咬合不良(Gear meshing)
- 往復式設備衝擊現象(Impact)
- 槽輪與固定器摩擦(Sheave against guard)

Unrelated Issues w/ Spiking

- 不平衡(Unbalance)
- 不對心(Misalignment)
- 軸彎曲(Bent shaft)
- 馬達電氣問題(Electrical problems)
- 偏心轉子(Eccentric rotor)
- 共振(Resonance)
- 結構鬆動(Structure looseness/Weakness)
- 拍擊問題(Beat vibration problem)

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Spike Energy Measurement

・量測需用相同的元件(同一感測器,安裝方式,信號線),否則量測數值會很大的變動
・儘量不要以手握頂桿方式量測,Spike Energy對頂住的力量大小很敏感
・建立一套自我設備Spike Energy的警戒表及基本頻譜
・量測時盡量維持運轉及操作條件一致

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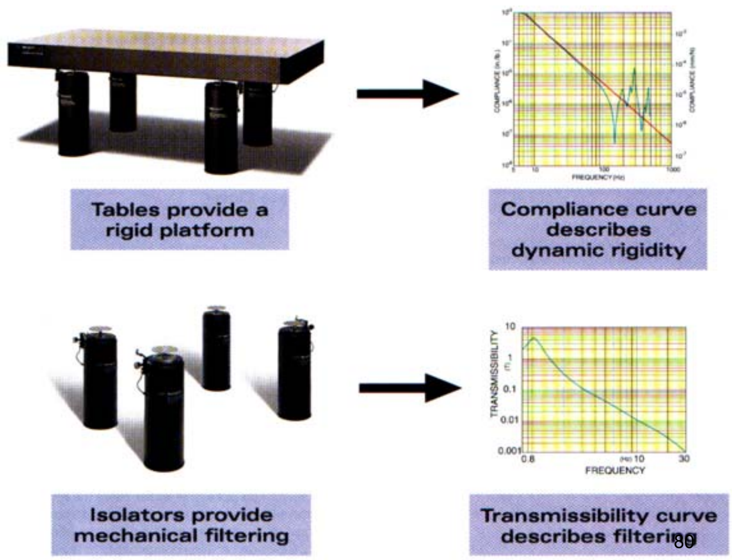
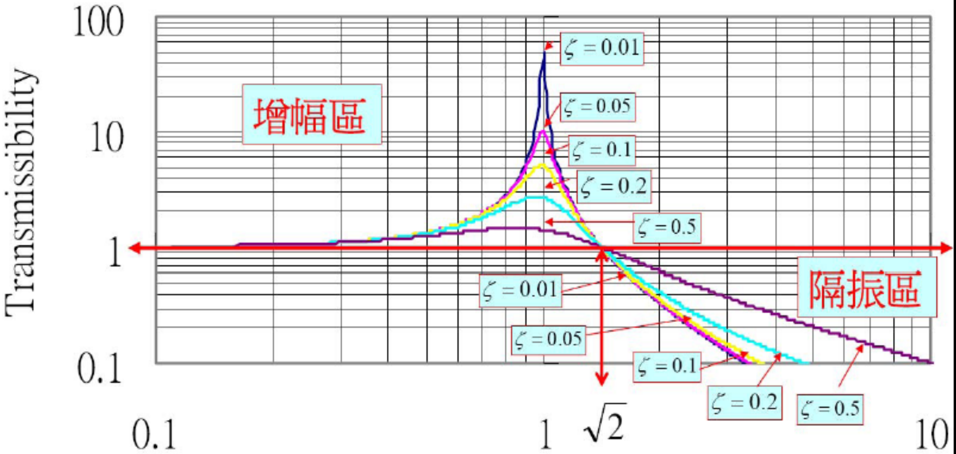
Part IV: Vibration Control

Optical Table

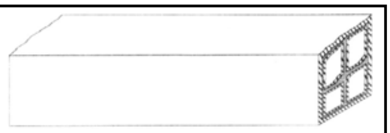
Vibration isolator & Optical Table

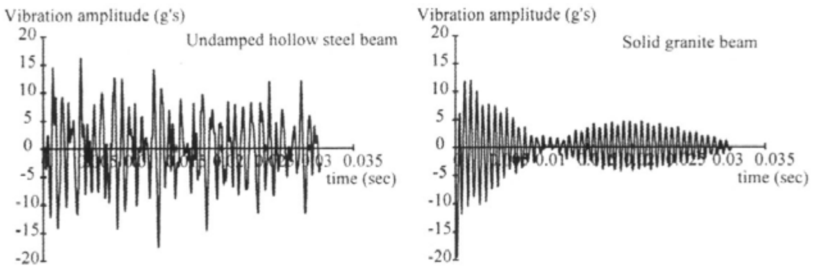


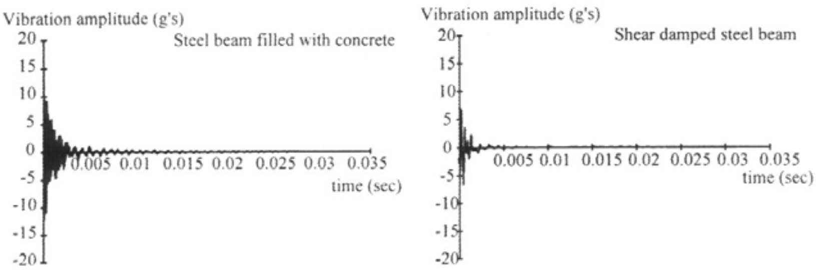
Transmissibility



Shear Dampers







成大機械

Active Vibration Control

- Adding feedback control
  - E.g., PD control, LQR optimal control
- Adding active elements
  - E.g., piezoelectric materials
- Adding external fields
  - E.g., magnetic field to interact with ferromagnetic materials

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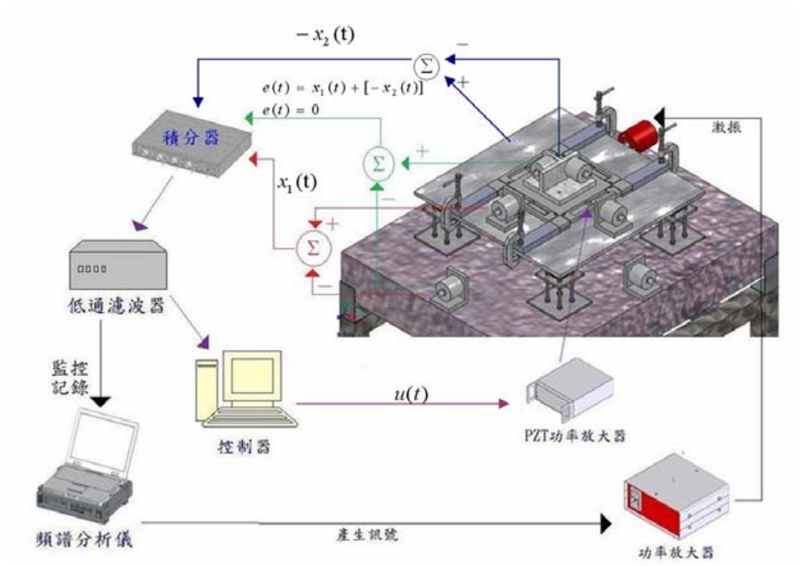
Active Vibration Control

實驗流程圖

(Flow)



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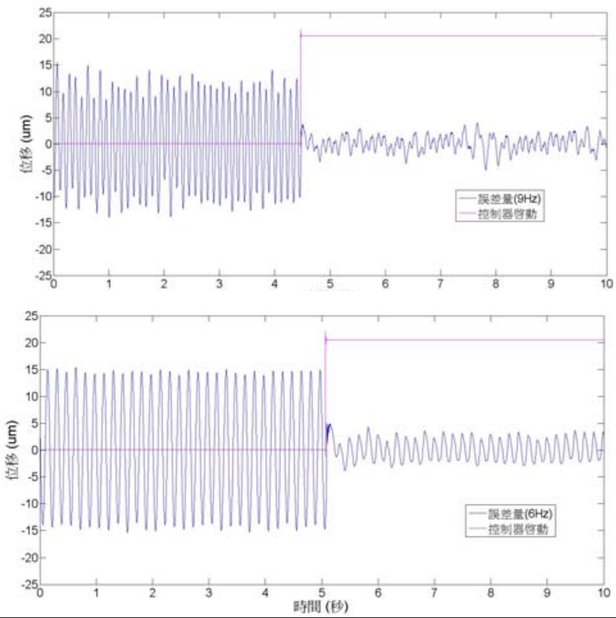
Typical Results



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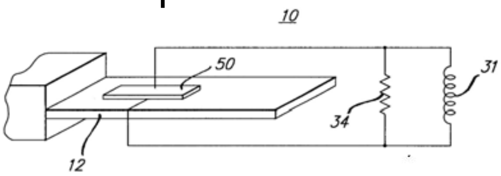
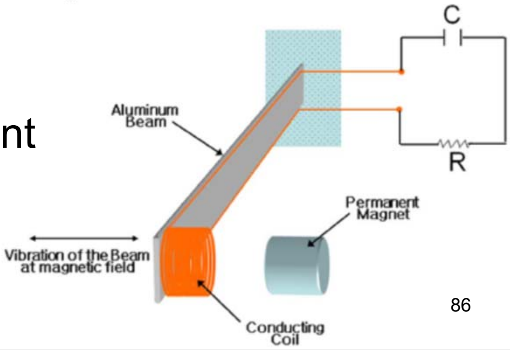
Active Materials and Structures

- Piezoelectric shunt circuit damper



- Electromagnetic shunt circuit damper



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